

W-3318**M.A./M.Sc. (Fourth Semester) Examination, June-2020****MATHEMATICS****Paper - 412****Special Functions****Time : Three Hours**

Maximum Marks : 85 (For Regular)

Minimum Pass Marks : 29

Maximum Marks : 100 (For Private)

Minimum Pass Marks : 34

Note : Attempt **all** questions.**Unit-I**

- Q.1. a) For a complex number Z (Z not an integer), Show that $\Gamma(Z)\Gamma(1-Z) = \frac{\pi}{\sin \pi Z}$.
 b) State and prove Gauss multiplication theorem.

Unit-II

- Q.2. a) If $\operatorname{Re}(c-a-b) > 0$ and c is neither zero nor a negative integer, then prove that-

$$F(a, b; c; 1) = \frac{\Gamma(c) \Gamma(c-a-b)}{\Gamma(c-a) \Gamma(c-b)}$$

- b) Prove that $\frac{d^n}{dx^n} [x^{a-1+n} F(a, b; c; x)] = (a)_n x^{a-1} F\left[\begin{matrix} a+n, b \\ c \end{matrix}; x\right]$

Unit-III

- Q.3. Define Bessel function. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of t^n in the expansion of $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ in the ascending and descending powers of t .

Unit-IV

- Q.4. a) Show that $\int_{-\infty}^{\infty} x^2 e^{-x} \{H_n(x)\}^2 dx = \sqrt{\pi} 2^n n! \left(n + \frac{1}{2}\right)$
 b) Prove that $H_n^1(x) = 2n H_{n-1}(x)$ for $n \geq 1$

Unit-V

- Q.5. a) Show that

$$(\rho_1 - 1)x E(\alpha_1, \dots, \alpha_p; \rho_1, \dots, \rho_q; x) = x E(\alpha_1, \dots, \alpha_p; \rho_1 - 1, \rho_2, \dots, \rho_q; x) + E(\alpha_1 + 1, \dots, \alpha_p + 1; \rho_1 + 1, \dots, \rho_q + 1; x)$$

- b) Prove that if $\operatorname{Re}(x) > 0, \operatorname{Re}(\gamma) > 0$, then

$$\int_0^{\infty} e^{-x\lambda} \lambda^{\gamma-1} {}_2F_1\left(\begin{matrix} \alpha, \beta \\ \delta \end{matrix}; -\lambda\right) d\lambda = \frac{\Gamma(\delta) x^{-\gamma}}{\Gamma(\alpha) \Gamma(\beta)} E(\alpha, \beta, \gamma; \delta; x)$$

