Roll No. ........................

# V – 3317

## M. A./M. Sc. (Fourth Semester) EXAMINATION, May/June, 2019

MATHEMATICS

Paper – 412

## SPECIAL FUNCTIONS

*Time : Three Hours*

*Maximum Marks : 85*

*Minimum Pass Marks : 29*

***Note–*** Attempt *all* questions.

1. Attempt any *five* parts– 5×5

(i) Prove that– $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

(ii) Show that for positive integral $n$,

$$B(P,n+1)=\frac{n!}{(P)_{n+1}}$$

(iii) Define Hypergeometric functions.

(iv) Show that–

P.T.O.





$${}_1F_1(a;b;z)=\frac{1}{\Gamma a}\int_0^{\infty}e^{-t}t^{a-1}\,{}_0F_1(-;b;zt)\,dt$$

(v) Define Legendre function.

(vi) Prove that–

$$P_n(-x)=(-1)^n P_n(x)$$

(vii) For Hermite polynomial show that–

$$H_n(-x)=(-1)^n H_n(x)$$

(viii) Define Laguerre polynomials and its generating function.

(ix) Write various recurrence relation of E-function.

(x) Write down relation between E-function and G-function.

### *Unit – I*

2. (a) State and prove Legendre Duplication formula.

(b) For a complex number $z$, show that–

$$\Gamma(z)\,\Gamma(1-z)=\pi/\sin\pi z$$

***Or***

(a) Prove that–

$$B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(b) Show that for $0\le m\le n$

$$(\alpha)_{n-m}=\frac{(-1)^m(\alpha)_n}{(1-\alpha-n)_m}$$

## Unit – II

3. Prove second contiguous function relation $(a-b)F=aF(a+\ )-bF(b+\ )$.

**Or**

State and prove Whipple's theorem.

## Unit – III

4. Prove that–

$$J_{\frac{1}{2}}(z)=\left[\frac{z}{\pi z}\right]^{1/2}\sin z$$

**Or**

Prove that–

$$J_0^2(x)-2\sum_{n=1}^{\infty}J_n^2(x)=1$$







## Unit – IV

5. Prove that– $H_{2n}(0)=(-1)^n\dfrac{(2n)!}{n!}.$

**Or**

Prove that–

$$\int_{-\infty}^{\infty}e^{-x^2}H_n(x)H_m(x)\,dx=\begin{cases}0, & \text{if } m\ne n\\ 2^n n!\sqrt{\pi}, & \text{if } m=n\end{cases}$$

## Unit – V

6. Show that–

$$\int_0^{\infty}e^{-y}y\,G_{p,q}^{m,n}\left[xy\Big/\begin{matrix}a_1,\ldots\ldots,a_p\\ b_1,\ldots\ldots,p_q\end{matrix}\right]dy$$

$$G_{p+1,q}^{m,n+1}\left(x\Big/\begin{matrix}\alpha,\ a_1\ldots\ldots\ldots,a_p\\ b_1\ldots\ldots\ldots\ldots,p_q\end{matrix}\right)$$

**Or**

Show that–

$$\alpha,\ x\,E(\alpha_1,\ldots\ldots\ldots\alpha_p;\beta_1,\ldots\ldots\ldots,\beta_q:x)$$

$$=xE(\alpha_1+1,\alpha_2,\ldots\ldots\alpha_p;\ \beta_1,\beta_2\ldots\ldots\beta_q:x)$$

$$+E(\alpha_1+1,\ldots\ldots\alpha_p+1:\beta_1+1,\ldots\ldots\beta_q+1:x)$$