Roll No. ....

Y – 5186

M.A./M.Sc. (Mathematics)
(First Semester)
EXAMINATION, Dec. 2021

Paper 103

INTEGRAL TRANSFORMS

Time : Three Hours

Maximum Marks : 85 (For Regular Students)
Minimum Pass Marks : 29

Maximum Marks : 100 (For Private Students)
Minimum Pass Marks : 34

Note : Attempt *All* questions.

1. Attempt any *five* questions from the following : 5×5=25/6×5=30

(i) Evaluate :

$$L\{F(t)\}$$

if :

$$F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$$

P.T.O.

(ii) If $L^{-1}\{f(p)\} = F(t)$ then prove that :

$$L^{-1}\{f(ap)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$$

(iii) Solve the following problem by means of the Laplace transform :

$$\frac{d^2x}{dt^2} + x = f(t)$$

if

$$x = x' = 0 \text{ for } t = 0$$

(iv) Solve the integral equation :

$$F(t) = 1 + \int_0^t F(u) \sin(t-u) du \cdot$$

(v) If $y(x, t)$ is a function of x and t then prove that :

$$L\left\{\frac{\partial y}{\partial t}\right\} = p\bar{y}(x, p) - y(x, 0)$$

where

$$L\{y(x, t)\} = \bar{y}\{x, p\}.$$

(vi) Write down one-dimensional heat equation and wave equation. Explain both also.

(vii) The Fourier transform of $f'(x)$ the derivative of $f(x)$ is $-ip\tilde{f}(p)$, where $\tilde{f}(p)$ is the Fourier transform of $f(x)$. Prove it.

(viii) Find the cosine transform of the function $f(x)$ if :

$$f(x)=\begin{cases}\cos x, & 0<x<a\\ 0, & x>a\end{cases}$$

(ix) If $f(x)$ is continuous and $f'(x)$ is sectionally continuous then prove that :

$$F_S\{f'(x)\} = -pF_C\{f(x)\}$$

$$(p = 1, 2, 3, ...)$$

(x) Solve :

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ x>0,\ t>0$$

P.T.O.

subject to the conditions :

(1) U = 0 when $x = 0, t > 0$

(2) $U = \begin{cases}1, & 0<x<1\\ 0, & x\geq 1\end{cases}$ when $t = 0$

and

(3) $U(x, t)$ is bounded.

Unit-I

2. Find : 12/14

$L\{\sin(\sqrt{t})\}$ and

$$L\left\{\frac{\cos(\sqrt{t})}{\sqrt{t}}\right\}$$

Or

Find $L^{-1}\left\{\frac{1}{\sqrt{p}\,(p-a)}\right\}$ by the convolution theorem and deduce the value of :

$$L^{-1}\left\{\frac{1}{p\sqrt{(p+a)}}\right\}.$$

Unit-II

3. Solve the following problem by means of the Laplace transform : 12/14

$$(D^2 + n^2)y = a\sin(nt + \alpha),$$

$$y = Dy = 0 \text{ when } t = 0.$$

Or

Solve the following problem by means of the Laplace transform :

$$ty'' + y' + 4ty = 0 \text{ if } y(0) = 3, y'(0) = 0$$

Unit-III

4. A semi-infinite solid $x > 0$ is initially at temperature zero. At time $t > 0$, a constant temperature $V_0 > 0$ is applied and maintained at the face $x = 0$. Find the temperature at any point of the solid at any time $t > 0$. 12/14

Or

A string is stretched between two fixed points (0, 0) and (c, 0). If it is displaced into the curve $y = b \sin\left(\frac{\pi x}{C}\right)$ and released from rest in that position at time $t = 0$, find its displacement at any time $t > 0$ and any point $0 < x < C$.

Unit-IV

5. Find the Fourier transform of : 12/14

$$F(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate :

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^2}\right) \cos\left(\frac{x}{2}\right) dx.$$

Or

Find the Fourier sine and cosine transforms of the function x^{m-1}.

Unit-V

6. Using the Fourier sine transform solve the partial differential equation :

$$\frac{\partial v}{\partial t} = k \frac{\partial^2 V}{\partial x^2} \text{ for } x > 0, t > 0,$$ under the boundary conditions $V = V_0$ when $x = 0$, $t > 0$ and the initial condition $V = 0$ when $t = 0, x > 0$. 12/14

Or

Solve the boundary value problem

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, U(0, t) = 1, U(\pi, t) = 3,$$

$$U(x, 0) = 1, \text{ where } 0 < x < \pi, t > 0.$$