Roll No. .....

# U – 5538

## M. A./M. Sc. (First Semester)

## EXAMINATION, Nov./Dec., 2018

MATHEMATICS

Paper – 103

## INTEGRAL TRANSFORMS

*Time : Three Hours*

*Maximum Marks : 85 (For Regular Students)*

*Minimum Pass Marks : 29*

*Maximum Marks : 100 (For Private Students)*

*Minimum Pass Marks : 34*

***Note–*** Attempt *all* questions.

1. Attempt any *five* from the following– ~~5×5=25~~

(i) Find L (sinh at) and L (cosh at).

(ii) If $L\{f(t)\} = f(s)$ then

$$\frac{1}{s} f(s) = L\{\int_0^t F(u)\,du\}.$$







(iii) Define inverse Laplace transform operator.

(iv) Using Laplace transform, solve $(D^2 - 2D - 2)y = 0$, where $y = Dy = 1$, when $t = 0$.

(v) Write two dimensional heat equation.

(vi) Write one dimensional wave equation.

(vii) Define Fourier transform.

(viii) Find finite sine transform of $e^{ax}$.

(ix) Write Laplace equation.

(x) Find the finite Fourier sine and cosine transform of $f(x) = 1$.

2. Find the Laplace transform of $\frac{\sin at}{t}$.

Does the transform of $\frac{\cos at}{t}$ exists. ~~12~~

***Or***

Evaluate $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$ by convolution theorem.

Using Laplace transform method, solve

$y''(t)+y(t)=t$, given that $y'(0)=1$, $y(\pi)=0$. 12

***Or***

(i) Apply Laplace transform to solve

$$\frac{d^2y}{dx^2}+y=6\cos 2t, \text{ given } y=3, \frac{dy}{dx}=1, \text{ when } t=0.$$ 6

(ii) Solve– 6

$$F'(t)=t+\int_0^t F(t-u)\cos u\, du, F(0)=4$$

4. Solve $\frac{\partial y}{\partial t}=3\frac{\partial^2 y}{\partial x^2}$

$$y\left(\frac{\pi}{2},t\right)=0,\ y(x,0)=30\cos 5x$$

$$\left(\frac{\partial y}{\partial x}\right)_{x=0}=0.$$ 12

***Or***

Find the solution of $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial t^2}=xt$







Where $u=\frac{\partial u}{\partial x}=0$, when $t=0$.

5. Find Fourier cosine transform of $e^{-x^2}$. 12

***Or***

Find the Fourier transform of–

$$f(x)=\begin{cases} x & , |x|\le a \\ 0 & |x|>a \end{cases}.$$

6. Use finite Fourier transform to solve

$$\frac{\partial u}{\partial t}=\frac{\partial^2 y}{\partial x^2},$$ given that

$u(0,t)=0, u(\pi,t)=0$

$u(x,0)=2x$, where $0<x<\pi, t>0$. 12

***Or***

Solve the following by use of Fourier transform :

$$\frac{\partial y}{\partial t}=2\frac{\partial^2 y}{\partial x^2},\ 0<x<4, t>0$$

with boundary conditions $u(0,t)=u(4,t)=0$ and $u(x,0)=2x$.