Roll No. ....

Y – 5184

M. A./M.Sc. Mathematics (First Semester) EXAMINATION, Dec.-2021

Paper-101

ADVANCED ABSTRACT ALGEBRA-I

Time : Three Hours

Maximum Marks : 85 (For Regular Students)

Minimum Pass Marks : 29

Maximum Marks : 100 (For Private Students)

Minimum Pass Marks : 34

Note– Attempt *all* questions.

1. Solve any *five* parts— 5×5=25/6×5=30
   (i) Define p-group and p-sylow subgroup.
   (ii) Define Normal and subnormal series.
   (iii) Define unital R-module.
   (iv) Define isomorphism of Modules
   (v) Define field extensions and degree of field extensions.

P.T.O.

   (vi) Define algebraic element and degree of an algebraic element.
   (vii) Define automorphism of a field.
   (viii) Define fixed field.
   (ix) Define invariant subspace.
   (x) Define Nilpotent Transformation and index of nilpotency.

2. State and prove Jordan Holder Theorem for finite groups. 12/14

Or

Define solvable group. Prove that a subgroup of a solvable group is solvable.

3. Define direct sum of submodules. State and prove necessary and sufficient condition for a module M to be the direct sum of its submodules M_1 and M_2. 12/14

Or

State and prove fundamental Theorem of homomorphism of modules.

4. Suppose K is an extension of a field F. Prove that the element $a \in k$ is algebraic over F if and only if $F(a)$ is a finite extension of F. 12/14

Or

Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

5. Suppose k is a finite extension of a field F. Then show that G(k, F) is a finite group and its order. 0(G, k(F)) satisfies the relation.

$$0\,[G(k, F)] \leq [k : F].$$ 12/14

Or

Suppose K is normal extension of F of characteristic zero and H is a subgroup of G(K, F). Let K_H be the fixed field of H. Then prove that

$$[K : K_H] = O(H)$$

6. If $T \in A(V)$ has all its characteristic roots in F then there is a basis in which the matrix of T is triangular. Prove it. 12/14

Or

Reduce the matrix $A = \begin{bmatrix} 5 & 1 & -2 & 4 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

into Jordan Canonical form.